HOMEWORK 5 – ANSWERS TO (MOST) PROBLEMS

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Section 3.4: The Chain Rule

3.4.12. $f'(t) = \cos(e^t)\, e^t + e^{\sin(t)} \cos(t)$

3.4.45. $y' = -\sin(\sqrt{\sin(\tan(\pi x))}) \frac{1}{2\sqrt{\sin(\tan(\pi x))}} \cos(\tan(\pi x)) \sec^2(\pi x) \pi$

3.4.49. $y' = \alpha e^{\alpha x} \sin(\beta x) + e^{\alpha x} \beta \cos(\beta x)$; $y'' = e^{\alpha x}((\alpha^2 - \beta^2)\sin(\beta x) + 2\alpha\beta\cos(\beta x))$

3.4.63.

(a) $h'(1) = f'(g(1))g'(1) = f'(2)g'(1) = 5 \times 6 = 30$
(b) $H'(1) = g'(f(1))f'(1) = g'(3)f'(1) = 9 \times 4 = 36$

3.4.66.

(a) $h'(2) = f'(f(2))f'(2) = f'(1)f'(2) = -\frac{1}{2} \times 1 = -\frac{1}{2}$
(b) $g'(2) = f'(4)4 = 1 \times 4 = 4$

3.4.67. $g'(3) = \frac{1}{2\sqrt{f(3)}} f'(3) = \frac{1}{2\sqrt{2}} \left(\frac{-2}{3}\right) = \frac{-1}{3\sqrt{2}}$ (which you can rewrite as $-\frac{\sqrt{2}}{6}$)

3.4.68.

(a) $F'(x) = f'(x^\alpha)\alpha x^{\alpha-1}$
(b) $G'(x) = \alpha \left(f(x)\right)^{\alpha-1} f'(x)$

3.4.72.

(a) $f'(x) = g(x^2) + xg'(x^2)2x = g(x^2) + 2x^2 g'(x^2)$
(b) $f''(x) = g'(x^2)(2x) + 4xg'(x^2) + 2x^2 g''(x^2)2x = 6xg'(x^2) + 4x^3 g''(x^2)$

3.4.80.

(a) $v(t) = s'(t) = -A\omega \sin(\omega t + \delta)$
(b) We want $v(t) = 0$, so $\omega t + \delta = \pi m$, so $t = \frac{\pi m - \delta}{\omega}$ (m is an integer)

3.4.85. $a(t) = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds} v(t)$

3.4.86.

(a) $\frac{dV}{dr}$ is the rate of change of V as the radius r changes, and $\frac{dV}{dt}$ is the rate of change of V as the time t changes
(b) $V(t) = \frac{4}{3}\pi r^3$, so $\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$

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Section 3.5: Implicit differentiation

3.5.3. $y' = -\frac{y^2}{x^2}$

3.5.19. $y' = \frac{e^y \sin(x) + \cos(xy) y}{e^y \cos(x) - x\cos(xy)}$

3.5.29. $y = x + \frac{1}{2}$

3.5.32. $y = -2$

3.5.44. See separate document 'Solution to 3.5.44'

Note: I would also suggest you (but you don't have to) to do 3.5.45 and 3.5.46. You can find solutions on the document 'Solutions to 3.5.44, 3.5.45, 3.5.46')

3.5.51. $\frac{2}{\sqrt{1-(2x+1)^2}}$ (or $\frac{1}{\sqrt{-(x^2+x)}}$, either answer is fine)

3.5.54. $\frac{1 - \frac{x}{\sqrt{1+x^2}}}{1 + (x - \sqrt{1+x^2})^2} = \frac{1}{2(1+x^2)}$

Note: This is a ridiculous simplification, and don't worry about this too much, but here are the steps:

(1) First put everything under a common denominator
(2) Then expand out the $1 + (x - \sqrt{1+x^2})^2$ in the denominator, and simplify and factor out the 2
(3) Then multiply the numerator and the denominator by $\sqrt{1+x^2} + x$ (conjugate form)
(4) Then expand out the $(\sqrt{1+x^2} + x)(1 + x^2 - x\sqrt{1+x^2})$-part of the denominator to get $\sqrt{1+x^2}$

3.5.63. Look up the handout 'Proof of the derivative of arccos' on my website!

3.5.77.

(a) $f(f^{-1}(x)) = x$, let $y = f^{-1}(x)$, then $f(y) = x$, so $f'(y)y' = 1$, so $y' = \frac{1}{f(y)} = \frac{1}{f(f^{-1}(x))}$
(b) $\frac{3}{2}$

Section 3.6: Derivatives of logarithmic functions

3.6.11. $g'(x) = \frac{1}{x\sqrt{x^2-1}} \left(\sqrt{x^2-1} + \frac{x^2}{\sqrt{x^2-1}} \right)$

3.6.21. $y' = 2\log_{10}(\sqrt{x}) + 2x \frac{1}{\ln(10)\sqrt{x}} \times \frac{1}{2\sqrt{x}} = 2\log_{10}(\sqrt{x}) + \frac{1}{\ln(10)}$

3.6.41. $y' = \sqrt{\frac{x-1}{x^4+1}} \left(\frac{1}{2(x-1)} + \frac{2x^3}{x^4+1} \right)$

3.6.44. $y' = x^{\cos(x)} \left(-\sin(x)\ln(x) + \frac{\cos(x)}{x} \right)$